



Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at <http://about.jstor.org/participate-jstor/individuals/early-journal-content>.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

THE AMERICAN JOURNAL

OF

SEMITIC LANGUAGES AND LITERATURES

VOLUME XXXVI

JULY 1920

NUMBER 4

A MATHEMATICAL CUNEIFORM TABLET

By H. F. LUTZ

University of Pennsylvania

CBS 8536, according to the Museum catalogue, was purchased from dealers, who bought the tablet from Arabs. The tablet was possibly unlawfully acquired by them at Nippur or at Abu Hatab. It is splendidly preserved and the writing is deeply cut and easily legible. The reddish-brown tablet measures 130 mm. (width) by 156 mm. (length). Column 1 contains the division of 60,¹ which it carries through, with the exception of 7 up to 10. It then proceeds up to 81, omitting a great number of intermittent divisors. Columns 2 to 12 contain tables of multiplication. The scheme is in every instance the same. It multiplies a number up to 20, followed thence by 30, 40, and 50. Then follows the multiplication of the number with its own self. With the exception of column 3, the succeeding line contains the square of the number of the preceding line. The

¹ Col. 1, l. 1, is not quite clear to me. It is a division, the result of which is 40. The sign, which I read *gal* with a question mark looks more like *bi*. The perpendicular stroke in the sign is not as clearly shown on the tablet as it is on my copy. It must, perhaps, be suppressed.

square is expressed by the sign, which is to be read according to other mathematical texts, which write the word much better than does this text, *ib-di*. Thus column 4, $2025 = 45^2$; column 6, $1600 = 40^2$; column 7, $1296 = 36^2$, etc. In column 2 the scribe has, by mistake, omitted the number 50, read 50^2 . The two last lines contain in each column the division of 3600 by the first number of the column, and that of its result, arriving thus in each instance at the number first given.

The importance of the tablet lies in its use of fractional numbers. Column 5 is most remarkable in that respect. It exhibits a very high state of arithmetical knowledge among the Babylonians, and it is particularly remarkable when we consider the age of the tablet. According to the general style and the writing of the tablet, it cannot be placed later than the Cassite period, but it seems more probable that it goes back even to the First Dynasty period, *ca.* 2000 B.C.

To those who are interested in the way the ancient Babylonians treated the fractions, but who are not acquainted with the fractional system of that people, a few remarks regarding the same seem to be necessary. The fractions are always expressed in terms of sixtieth, or rather three hundred-sixtieth, which were reduced to the simplest fractions, whenever the numerator could be expressed either by the number 1, or else by a number that was one short of the number of the denominator. That is, $\frac{60}{360} = \frac{10}{60} = \frac{1}{6}$ (*šuššu* or *šussu*), $\frac{120}{360} = \frac{20}{60} = \frac{1}{3}$ (*šuššān*), $\frac{180}{360} = \frac{30}{60} = \frac{1}{2}$ (*šunu*), $\frac{240}{360} = \frac{40}{60} = \frac{2}{3}$ (*šinipu*), $\frac{300}{360} = \frac{50}{60} = \frac{5}{6}$ (*pārab*), $\frac{90}{360} = \frac{15}{60} = \frac{1}{4}$ (*rubcu*), $\frac{72}{360} = \frac{12}{60} = \frac{1}{5}$, etc. In order to express $\frac{1}{9}$ they were compelled to make use of a fraction $\frac{6+40}{60} = \frac{6+4}{60}$. Since the Babylonians could not express a fraction in which the numerator was higher than one and two numbers lower than the whole (i.e., $\frac{2}{7}$, $\frac{4}{7}$, $\frac{5}{8}$, $\frac{7}{9}$) they operated in a manner shown in the following example, which is taken from the text (see col. 1, last line: $60+81 = \frac{20}{27}$). In order to express the equivalent of our $\frac{20}{27}$ they were compelled to make use of the fraction:

$$\frac{44\frac{30}{60} + \frac{6+40}{60}}{60} = \frac{44\frac{26+40}{60}}{60} = \frac{44\frac{4+4}{60}}{60}.$$

That is,

$$\frac{60}{81} = \frac{20}{27} \times \frac{60}{60} = \frac{400}{540} = \frac{44\frac{4}{60}}{60} = \frac{44\frac{4+4}{60}}{60}.$$

In column 5 we have a multiplication of $44\frac{1}{3} + \frac{1}{9} \times 44\frac{1}{3} + \frac{1}{9}$, the result of which is given correctly as:

$$1975 \frac{18 + \frac{31 + \frac{60}{60}}{60}}{60}.$$

That is,

$$44\frac{4}{9} \times 44\frac{4}{9} = \frac{400}{9} \times \frac{400}{9} = \frac{160000}{81} = 1975\frac{25}{81}.$$

Column 1	Column 2	Column 3
... gal(?) -bi 40-ām	1 50	1 48
šu a-na gal-bi 30-ām*	2 100	2 96
igi 2 30	3 150	3 144
igi 3 20	4 200	4 192
igi 4 15	5 250	5 240
igi 5 12	6 300	6 288
igi 6 10	7 350	7 336
igi 8† 7½	8 400	8 384
igi 9 6⅔	9 450	9 432
igi 10 6	10 500	10 480
igi 12 5	11 550	11 528
igi 15 4	12 600	12 576
igi 16 3⅔	13 650	13 624
igi 18 3¼	14 700	14 672
igi 20 3	15 750	15 720
igi 24 2½	16 800	16 768
igi 25 2⅔	17 850	17 816
igi 27‡ 2⅔ + ⅓	18 900	18 864
igi 30 2	19 950	19 912
igi 35 1 52½	20 1000	20 960
	30 1500	30 1440
	40 2000	40 1920
igi 36 1⅔	50 a-na ¶ 50 2500	50 2400
igi 40 1½	2500 = 50²	48 a-na 48 2304
igi 45 1¼	igi 50 72	igi 48 75
igi 48 1¼	igi 72 50	igi 75 48
igi 50 1⅓		
igi 54 1⅓		
igi 60 1		
igi 64 ⅔ (1)		
igi 72 ⅔		
igi 80 ⅔		
igi 81 44⅓ + ⅓		
	60	

* $60 \div 2 = 30$; this is the meaning; the reading of *gal* is, however, uncertain. $\check{S}u = \check{S}u\check{S}u$, see K. 4378, (D. 88) 15 and Br. 10075.

† Number 7 omitted in tablet.

‡ Tablet reads 28, which must be a mistake. The number demanded by the result is 27.

§ $\frac{1}{9} + \frac{1}{9} = \frac{13}{60}$.

|| 35 is a mistake. It should be 32.

¶ The text has throughout an interesting variant of the more common *a-ra*.

Column 4			Column 5			Column 6		
1	45			$44\frac{1}{2}+\frac{1}{2}$		1	40	
2	90		1	$44\frac{1}{2}+\frac{1}{2}$		2	80	
3	135		2	88§		3	120	
4	180		3	133‡		4	160	
5	225		4	$177\frac{1}{2}+\frac{1}{2}†$		5	200	
6	270		5	$222\frac{1}{2}+\frac{1}{2}$		6	240	
7	315		6	266‡		7	280	
8	360		7	$361\frac{1}{2}$		8	320	
9	405		8	400		9	360	
10	450		10	$444\frac{1}{2}+\frac{1}{2}$		10	400	
11	495		11	488§		11	440	
12	540		12	533‡		12	480	
13	585		13	$577\frac{1}{2}+\frac{1}{2}†$		13	520	
14	630		14	$622\frac{1}{2}+\frac{1}{2}$		14	560	
15	675		15	$666\frac{1}{2}$		15	600	
16	720		16	$711\frac{1}{2}§$		16	640	
17	765		17	755		17	680	
18	810		18	800		18	720	
19	855		19	$844\frac{1}{2}+\frac{1}{2}$		19	760	
20	900		20	888§		20	800	
30	1350		30	1333‡		30	1200	
40	1800		40	$1777\frac{1}{2}+\frac{1}{2}$		40	1600	
50	2250		50	$2222\frac{1}{2}+\frac{1}{2}$		50	2000	
45 a-na 45			$44\frac{1}{2}+\frac{1}{2}$ a-na	$44\frac{1}{2}+\frac{1}{2}$		40 a-na 40	1600	
2025	45 ²			$6+\frac{1}{2}§$		1600	40 ²	
igi	45	80		$31+\frac{60}{60}$		igi	40	90
igi	80	45*	1975	$18+\frac{60}{60}$		igi	90	40
<hr/>								
$(44\frac{1}{2}+\frac{1}{2})^2$								
<hr/>								

Column 7			Column 8			Column 9		
15	540		15	450		15	375	
16	576		16	480		16	400	
17	612		17	510		17	425	
18	648		18	540		18	450	
19	684		19	570		19	475	
20	720		20	600		20	500	
30	1080		30	900		30	750†	
40	1440		40	1200		40	1000	
50	1800		50	1500		50	1250‡	
36 a-na 36	1296		30 a-na 30	900		25 a-na 25		
1296	36²		900	30²		625	35§	
igi	36	100	igi	30	120	25²		
igi	100	36	igi	120	30	igi	25	144
						igi	144	25

† Text wrongly 1110.

‡ Text wrongly 355.

§ The scribe wrote by mistake 35, and after seeing his mistake, placed the correct number 25 on the next line.

Column 10			Column 11			Column 12		
1	24		1	22½		1	20	
2	48		2	45		2	40	
3	72		3	67½		3	60	
4	96		4	90		4	80	
5	120		5	112½		5	100	
6	144		6	135		6	120	
7	168		7	157½		7	140	
8	192		8	180		8	160	
9	216		9	202½†		9	180	
10	240		10	225		10	200	
11	264		11	247½		11	220	
12	288		12	270		12	240	
13	312		13	292½		13	260	
14	336*		14	315		14	280	
15	360		15	337½		15	300	
16	384		16	360		16	320	
17	408		17	382½		17	340	
18	432		18	405		18	360	
19	456		19	427½		19	380	
20	480		20	450		20	400	
30	720		30	675		30	600	
40	960		40	900		40	800	
50	1200		50	1125		50	1000	
24 a-na 24			22½ a-na 22½			20 a-na 20		
576	576		506½			566½§		
24²			506½			386½§		
igi	24	150	22½²			20²		
igi	150	24	igi 22½	160		igi	20	180
			igi 160	22½		igi	180	20

* 6, i.e., 360 mistake of scribe for 5=300.

† Notice the interesting variant of the writing of ½.

§ The numbers can also be read 560½ and 380½. Their signification I fail to understand. We should expect 400 in each case.

Col. 1		OBVERSE		Col. 3
Col. 2		Col. 2		
				
				
				
				
				
				
				
				
				
				
				
				
				
				
				
				
				
				
				
				
				
				
				
				
				

